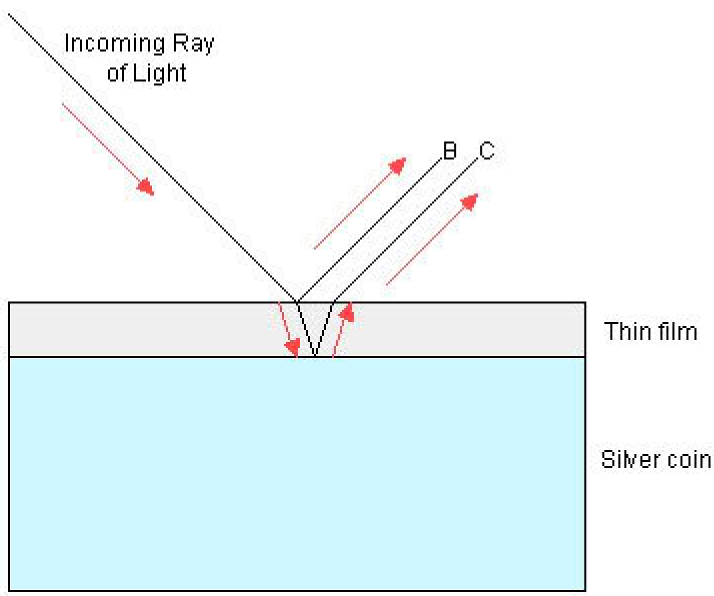
Physics of the Toning Progression of Rainbow Morgans

Nobody knows toning better than Doug Kurz, owner of **Sunnywood's "Somewhere Over the Rainbow" Toned Morgans**. This is the first and only (that I know about) complete set of Rainbow Toned Morgan dollars. This article delves into the physics of Thin Film Interference and how the human eye perceives the colors radiating off the surface of a nicely patinated Toned Morgan dollar. Put your thinking caps on and pull out those old high-school physics textbooks and enjoy! I did not edit this article much because if it ain't broke, don't fix it!

Several articles have been devoted in the past to a discussion of the thin film interference phenomenon, which is responsible for the colorful toning we see on silver coins. I would like to delve more deeply into the physics, to see if we can develop a model for understanding and predicting the actual "standard progression" of colors that we observe on silver coins.

For example, why does toning start out light gold? Why does it then progress to gold, amber, russet, burgundy, cobalt blue, light blue, lemon yellow, orange, red, magenta, blue, blue-green, emerald green, and so on. Why can't a coin have gold toning that progresses to green, or some other color, rather than following the "standard progression?"

It turns out that once you understand the thin film interference mechanism, it is possible to model the colors that we actually observe on the coins. As has been shown before, the light reflecting off the surface of a toned coin looks something like this:



Some of the incoming light reflects off the upper surface of the toning layer, and exits as ray B. However, a portion of the light travels through the thin film layer, and then reflects off the denser metal surface of the coin underneath the toning layer, and exits as ray C. Notice that the light in ray C has to travel a longer distance than ray B. The extra distance (d) traveled is approximately twice the thickness of the toning layer (t). [More accurately, d=2t/sin(x), where x is the angle of incidence of the light hitting the coin's surface below the toning. For light coming in at a direct perpendicular to the surface, x=90°, sin(x)=1, and d=2t.]

Because ray C travels the extra distance, the emerging reflected light of ray C is phase-shifted relative to ray B. If the extra distance traveled by ray C is precisely equal to the wavelength of the light (or a multiple thereof), then the two re-united beams B and C will be in phase, and there will be constructive interference, or reinforcement, of the luminosity at that wavelength. If the extra distance d is precisely equal to one-half the wavelength of the light (or an odd multiple thereof), then the two re-united beams will be precisely out of phase, and there will be destructive interference, or cancellation, of the light at that wavelength. For intermediate phase-shifts, there will be moderate cancellation or reinforcement.

(I am leaving out more details, such as one-half wavelength phase shifts due to differences in the index of refraction of air vs. the toning layer.)

So what we have so far, is that for a given thickness of toning layer, and a given wavelength of light, there will be either constructive interference (reinforcement), or destructive interference (cancellation) of the light as a result of the toning layer. As the toning layer thickness increases, the beams B and C go in and out of phase.

The next step is to understand how we perceive light. Visible light covers a spectrum of wavelengths from about 400 nanometers (nm) for violet to about 700 nm for deepest red. A rainbow occurs when full-spectrum light (i.e. light containing all wavelengths in the visible spectrum) is refracted by a prism, and separated out into all the different wavelengths. For each color in the spectrum, there is a corresponding wavelength of light. However, the way we perceive light is peculiar to the design of our retina, with its "rods" and "cones" (the photoreceptors). The cones perceive color, and there are three types of cones in the human eye. Each one has a peak sensitivity at a particular wavelength:

The three types of cones are B=blue (peak sensitivity at about 440 nm), G=green (535 nm), and R=red (575 nm). The peak sensitivity wavelengths vary in different sources and papers on the subject, but you get the idea.

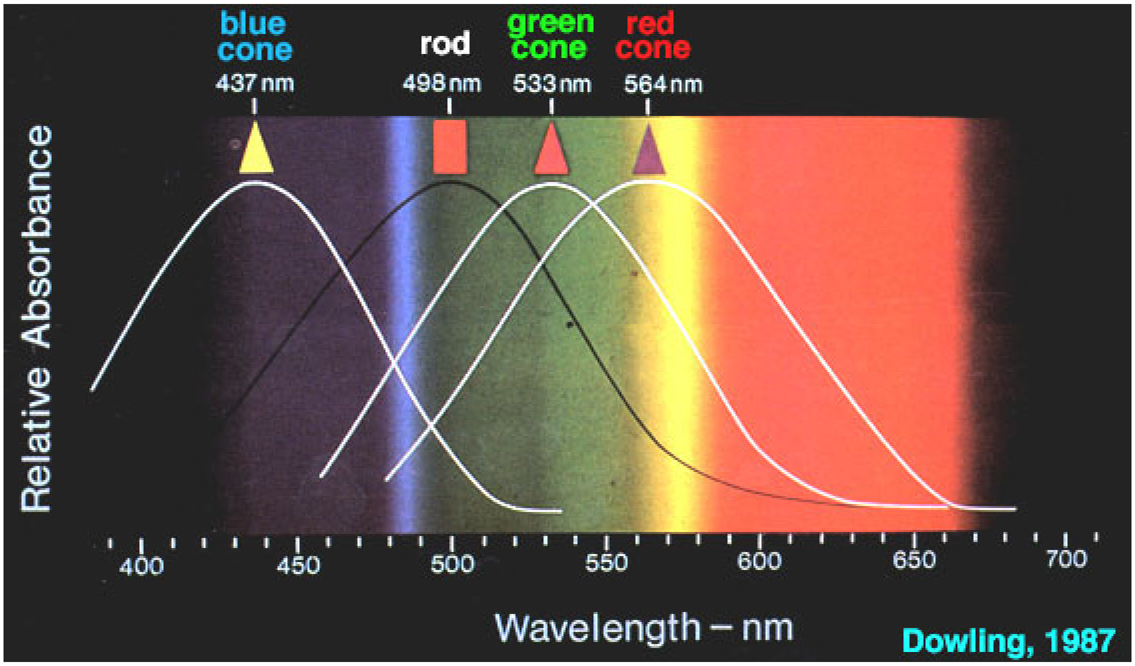


Fig. 14. The peak spectral sensitivities of the the 3 cone types and the the rods in the primate retina (Brown and Wald, 1963). From Dowling's book (1987).

So for example, when our eye sees yellow light, we know it is yellow because of the way our red and green cones respond to it, and our brain translates that into a perception of yellow. The peculiar consequence of our having three specific cone receptors, rather than one that covers the entire spectrum, is that you can "trick" the human eye. If the incoming light contains only two wavelengths - red and green - the eye will perceive it as yellow, just as though the incoming light were at the wavelength for yellow. So I write: G+R=Y. Similarly, blue light plus green light will be perceived as cyan (B+G=C), and blue light plus red light will be perceived as magenta (B+R=M). The combination of all three is perceived as white: B+G+R=W.

Because of this, if you take full-spectrum light, and remove all the blue wavelengths, that means only the red and green cones can respond, and the eye sees yellow. This is called the "subtractive" property of light, which refers not to the light itself, but the way we perceive it. Yellow paint, for example, actually looks yellow because it contains pigments that absorb blue light. We can represent this algebraically as follows: W-B = (B+G+R)-B = G+R = Y

So, if the toning layer has a thickness that results in the cancellation of blue light, then the coin will appear yellow. Depending on which wavelengths get destructively cancelled or constructively reinforced at a particular toning layer thickness, the exit light will have some combination of components of light that appears white (R+G+B), yellow (R+G), cyan (B+G), magenta (B+R), blue (B), green (G), and red (R).

Example: if the incoming light has 4 units each of B, G, R; but the exit light after interference has

3 units B, 2 units G, and 1 unit R, what will we see? We will see a combination of one unit white (B+G+R), one unit cyan (B+G), and one unit blue (B). The result will be a light cyan-blue.

If the exit light contains four units B, five units G, four units R, we will see four units white (B+G+R), plus one leftover units of green. This will give us a bright light green.

If the exit light contains one unit B, five units G, one unit R, we will see one unit white (B+G+R), plus four leftover units of green. This will give us a stronger deeper (less bright) green.

In this way, I can model and predict the color of the exit light. [For the technical among you, the model depends on choosing the right peak wavelengths to represent blue, green and red as seen by our cone photoreceptors. There is some latitude in the numbers chosen, particularly for red; I'm not sure whether the peak sensitivity of 575nm is the right choice, or perhaps a higher number up to 660nm. For our purposes I am assuming the angle of incidence "x" equals 90°, and I am ignoring the half-wavelength shift off the upper surface due to the refractive index change, which actually would imply that light gold toning becomes visible at even lower layer thicknesses.] Here then is a sample spreadsheet:

(nm) Film Thickness	(nm) Extra Distance	Constructive - Destructive Oscillator 445 Blue	535 Green	575 Red	Total Luminosity	B+G+R=W White	B+G=C Cyan	B+R=M Magenta	G+R=Y Yellow	Residual Blue	Residual Green	Residual Red	Observed Color
100	200	0.1011	0.2523	0.3043	0.6578	0.1011	0	0	0.1512129	0	0	0.0520114	Light Gold
110	220	0.0112	0.1776	0.2348	0.4236	0.0112	0	0	0.1663341	0	0	0.0572125	Amber
120	240	0.0787	0.1028	0.1652	0.3467	0.0787	0	0	0.0241521	0	0	0.0624137	Russet
130	260	0.1685	0.0280	0.0957	0.2922	0.0280	0	0.0676148	0	0.0728872	0	0	Burgundy
140	280	0.2584	0.0467	0.0261	0.3312	0.0261	0.020642	0	0	0.211698	0	0	Cobalt Blue
150	300	0.3483	0.1215	0.0435	0.5133	0.0435	0.0780171	0	0	0.2268193	0	0	Cobalt Blue
160	320	0.4382	0.1963	0.1130	0.7475	0.1130	0.0832182	0	0	0.2419406	0	0	Cobalt Blue
170	340	0.5281	0.2710	0.1826	0.9817	0.1826	0.0884193	0	0	0.2570619	0	0	Cobalt Blue
180	360	0.6180	0.3458	0.2522	1.2159	0.2522	0.0936205	0	0	0.2721831	0	0	Cobalt Blue
190	380	0.7079	0.4206	0.3217	1.4502	0.3217	0.0988216	0	0	0.2873044	0	0	Cobalt Blue
200	400	0.7978	0.4953	0.3913	1.6844	0.3913	0.1040228	0	0	0.3024257	0	0	Cobalt Blue
210	420	0.8876	0.5701	0.4609	1.9186	0.4609	0.1092239	0	0	0.317547	0	0	Cobalt Blue
220	440	0.9775	0.6449	0.5304	2.1528	0.5304	0.114425	0	0	0.3326683	0	0	Cobalt Blue
230	460	0.9326	0.7196	0.6000	2.2522	0.6000	0.1196262	0	0	0.2129581	0	0	Light Blue
240	480	0.8427	0.7944	0.6696	2.3067	0.6696	0.1248273	0	0	0.0483041	0	0	Cyan
250	500	0.7528	0.8692	0.7391	2.3611	0.7391	0.0136786	0	0	0	0.1163499	0	Pale Mint Green
260	520	0.6629	0.9439	0.8087	2.4155	0.6629	0	0	0.1457743	0	0.1352296	0	Pale Mint Green
270	540	0.5730	0.9813	0.8783	2.4326	0.5730	0	0	0.3052272	0	0.1030475	0	Greenish Yellow
280	560	0.4831	0.9065	0.9478	2.3375	0.4831	0	0	0.423396	0	0	0.041284	Yellow
290	580	0.3933	0.8318	0.9826	2.2076	0.3933	0	0	0.4385173	0	0	0.150833	Sunny Yellow
300	600	0.3034	0.7570	0.9130	1.9734	0.3034	0	0	0.4536386	0	0	0.1560341	Sunny Yellow
310	620	0.2135	0.6822	0.8435	1.7392	0.2135	0	0	0.4687598	0	0	0.1612353	Sunny Yellow
320	640	0.1236	0.6075	0.7739	1.5050	0.1236	0	0	0.4838811	0	0	0.1664364	Sunset Yellow
330	660	0.0337	0.5327	0.7043	1.2708	0.0337	0	0	0.4990024	0	0	0.1716375	Sunset Yellow
340	680	0.0562	0.4579	0.6348	1.1489	0.0562	0	0	0.4017641	0	0	0.1768387	Orange
350	700	0.1461	0.3832	0.5652	1.0945	0.1461	0	0	0.2371102	0	0	0.1820398	Orange
360	720	0.2360	0.3084	0.4957	1.0400	0.2360	0	0	0.0724562	0	0	0.187241	Red
370	740	0.3258	0.2336	0.4261	0.9856	0.2336	0	0.0921978	0	0	0	0.1002443	Red-Magenta
380	760	0.4157	0.1589	0.3565	0.9311	0.1589	0	0.1976432	0	0.0592086	0	0	Magenta
390	780	0.5056	0.0841	0.2870	0.8767	0.0841	0	0.2028444	0	0.2186615	0	0	Magenta-Blue
400	800	0.5955	0.0093	0.2174	0.8222	0.0093	0	0.2080455	0	0.3781143	0	0	Blue-Purple
410	820	0.6854	0.0654	0.1478	0.8986	0.0654	0	0.0824055	0	0.5375672	0	0	Blue
420	840	0.7753	0.1402	0.0783	0.9937	0.0783	0.061926	0	0	0.635094	0	0	Blue
430	860	0.8652	0.2150	0.0087	1.0888	0.0087	0.2062576	0	0	0.6502153	0	0	Blue
440	880	0.9551	0.2897	0.0609	1.3056	0.0609	0.2288501	0	0	0.6653366	0	0	Blue
450	900	0.9551	0.3645	0.1304	1.4500	0.1304	0.2340512	0	0	0.5905702	0	0	Light Blue
460	920	0.8652	0.4393	0.2000	1.5044	0.2000	0.2392523	0	0	0.4259162	0	0	Light Blue
470	940	0.7753	0.5140	0.2696	1.5589	0.2696	0.2444535	0	0	0.2612622	0	0	Cyan
480	960	0.6854	0.5888	0.3391	1.6133	0.3391	0.2496546	0	0	0.0966082	0	0	Cyan
490	980	0.5955	0.6636	0.4087	1.6678	0.4087	0.18681	0	0	0	0.0680458	0	Cyan Green
500	1000	0.5056	0.7383	0.4783	1.7222	0.4783	0.0273571	0	0	0	0.2326998	0	Light Cyan Emerald Gre
510	1020	0.4157	0.8131	0.5478	1.7766	0.4157	0	0	0.1320957	0	0.265258	0	Light Emerald Green
520	1040	0.3258	0.8879	0.6174	1.8311	0.3258	0	0	0.2915486	0	0.2704592	0	Emerald Green
530	1060	0.2360	0.9626	0.6870	1.8855	0.2360	0	0	0.4510015	0	0.2756603	0	Emerald Green

(Click on the picture to view the full size non blurry example - Opens in new window)

It still requires some tweaking, but this approach is on the right track, and basically predicts what we see on the coins. So for example, why does toning start out in the gold-amber category, before we see any blue? Because blue light has the shortest wavelength of the three cones, at 440 nm. That means the phase shift to cancel blue light is about 220 nm, which requires a toning layer of about 110 nm (ignoring the angle x mentioned above). So the very thinnest toning layers cancel blue before they cancel anything else, and when you cancel blue, you are left with R+G=Y, so toning always begins with faint yellow (gold), deepening to amber, russet, and burgundy, before we suddenly get blue, as predicted in the spreadsheet.

This is not quite complete or perfect, but I hope it helps to understand the origin of the magnificent colors we see on toned silver coins, such as this one:

(Photography by Brandon Kelley)